

## Problem 11

Use equation (1.8) to find the fractions that are equivalent to the following repeating decimals:

$$0.678571428571428571 \dots$$

### Solution

Observe the repeating pattern  $\dots 428571 \dots$ . The 67 at the beginning is anomalous.

$$0.678571428571428571 \dots = 0.25 + 0.428571 + 0.000000428571 + \dots$$

$$= \frac{1}{4} + \frac{428\,571}{10^6} + \frac{428\,571}{10^{12}} + \dots$$

$$= \frac{1}{4} + \sum_{i=1}^{\infty} \frac{428\,571}{10^{6i}}$$

$$= \frac{1}{4} + \sum_{i=1}^{\infty} \frac{428\,571}{1\,000\,000^i}$$

$$= \frac{1}{4} + 428\,571 \sum_{i=1}^{\infty} \frac{1}{1\,000\,000^i}$$

$$= \frac{1}{4} + 428\,571 \sum_{i=1}^{\infty} \left( \frac{1}{1\,000\,000} \right)^i$$

$$= \frac{1}{4} + 428\,571 \left[ -1 + \sum_{i=0}^{\infty} \left( \frac{1}{1\,000\,000} \right)^i \right]$$

$$= \frac{1}{4} + 428\,571 \left[ -1 + \frac{1}{1 - \left( \frac{1}{1\,000\,000} \right)} \right]$$

$$= \frac{1}{4} + 428\,571 \left( -1 + \frac{1\,000\,000}{999\,999} \right)$$

$$= \frac{1}{4} + 428\,571 \left( \frac{1}{999\,999} \right)$$

$$= \frac{19}{28}$$